DOCUMENT RESUME

ED 227 003 SE 040 918

AUTHOR Quintero, Ana Helvia
TITLE The Role of Conceptual Understanding in Solving Word Problems: Two-Step Problems.
PUB DATE 83
NOTE 26p.; Paper presented at the meeting of the Educational Research Association (Montreal, Canada, April, 1983).
PUB TYPE Speeches/Conference Papers (150) -- Reports - Research/Technical (143)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Conference Papers; Educational Research; Elementary Education; *Elementary School Mathematics; *Error Patterns; *Mathematical Concepts; *Mathematics Instruction; *Problem Solving; *Ratios (Mathematics)
IDENTIFIERS *Mathematics Education Research

ABSTRACT

This work studies the difficulties children have solving two-step word problems of a given structure (e.g., 7 lollypops/2 chocolate bars X 6 chocolate bars). Thirty-six children between the ages of 9 and 14 years were individually observed solving problems of the above type. Other tasks, repetition of the problem and model recognition, were used to probe children's representation of the problem. Analysis of children's errors on these tasks show that children's difficulties with this type of problems stem from different sources. The major ones were: understanding relationships such as the one contained in the statement "7 lollypops for every 2 chocolate bars"; given a ratio finding two new quantities that have the same ratio; and using two steps to solve the problems. These difficulties are not common to all word problems. Thus, it is important to have a clear description of the types of problems that children are asked to solve in order to uncover the particular sources of difficulty. (Author)

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The Role of Conceptual Understanding in Solving Word Problems: Two-Step Problems

By:

Ana Helvia Quintero
Department of Mathematics
University of Puerto Rico
Río Piedras, Puerto Rico 00931

Paper accepted for presentation at the meeting of the Educational Research Association, Montreal, Canada, April 1983

SE040918

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The Role of Conceptual Understanding in Solving Word Problems: Two-Step Problems

Abstract

This work studies the difficulties children have solving two-step word problems of a given structure (eg., 7 lollypops/2 chocolate bars X 6 chocolate bars). Thirty-six children between the ages of 9 and 14 years were individually observed solving problems of the above type. Other tasks, repetition of the problem and model recognition, were used to probe children's representation of the problem.

Analysis of children's errors on these tasks show that children's difficulties with this type of problems stem from different sources. The major ones were: understanding relationships such as the one contained in the statement "7 lollypops for every 2 chocolate bars"; given a ratio finding two new quantities that have the same ratio; and using two steps to solve the problems.

These difficulties are not common to all word problems. Thus, it is important to have a clear description of the types of problems that children are asked to solve in order to uncover the particular sources of difficulty.

The Role of Conceptual Understanding in Solving Word Problems: Two-Step Problems

In the elementary grades word problems that can not be solved by a routine application of a single arithmetic operation cause a great deal of difficulty (see National Assessment of Educational Progress 1979; Carpenter, Corbitt, Kepner, Lindquist and Reys, 1980).

The majority of the studies that try to determine children's difficulty with multi-step problems use categories for characterizing the problems that are very broad and frequently not informative (eg. "standarized word problem test"). For understanding children's behavior in solving word problems, there is a need to have a clear description of the types of problems that children are asked to solve. In this description it is very important to take into consideration the semantic aspect of the problem: that is the meaning of the concepts and relationships involved in the problems. The work done by Carpenter, Hiebert and Moser (1979), Carpenter and Moser (1982), Greeno (1980), Moser (1979) and Nesher (1982), on one-step addition and subtraction word problems, and the work of Quintero (1980, 1981), on one-step multiplication word problems, have shown that the concepts and relationships involved in the problem are a strong determiner of the problem difficulty for the children.

In the above research work cited the problems solved by the subjects were one-step problems. Once the concepts and relationships involved in a one-step problem are understood, the child has only to choose the

appropiate arithmetic operation to solve it. In multi-steps arithmetic problems besides choosing the operations to apply one has to plan and organize the order of applying the operations, and to which pair of numbers should each operation be applied. This may introduce a source of difficulty in multi-steps problems not present in one-step problems.

Students' difficulties solving multi-steps problems, then, can be due to difficulties in understanding the concepts and relationships involved in the problem; and/or difficulties in planning and organizing the method of solution. The present study intends to probe the role of each of these factors in children's performance when solving two-step problems of a given structure.

Method

Task Analysis

The problems used in the study were of the following type:

There are a x's for every b y's.

There are c y's.

How many x's are there?

For example:

"In the store they give 6 lollypops for every 2 chocolate bars you buy.

John bought 10 chocolate bars.

How many lollypops did they give him?"

Understanding the mathematical relationship contained in the statement "6 lollypops for every 2 chocolate bars" is a fundamental requirement for solving the problem. This relationship consists of a ratio. Besides understanding the given ratio to solve the above

problem one must understand that one can change the quantities that enter to form the ratio and still maintain the same ratio.

Once the above concepts and relationships are understood there are two possible methods by which the problem can be solved: One is to simplify first the ratio "6 lollypops for 2 chocolate bars" to "3 lollypops for chocolate bar" (i.e. find a/b) and then multiply that number by the number of chocolate bars bought, 3 X 10 (i.e. $\frac{a}{b}$ X c); or first find how many groups of 2 chocolate bars there are in 10 chocolate bars (i.e. find c/b) and then multiply that number by the number of "lollypops for 2 chocolate bars", 5 x 6 (i.e. c/b x a).

The nature of the numbers used in the problems could affect the method used by the children. For example, if in the above problem among the numbers in the statement "6 lollypops for every 2 chocolate bars, and bought 7 chocolate bars", only a/b is an integer, then it is expected that children will first simplify the ratio; 6/2 = 3 lollypops for 1 chocolate bar and then multiply that quantity by 7. On the other hand, if the following numbers are used: "7 lollypops for every 2 chocolate bars and bought 6 chocolate bars"; that is, only c/b is an integer, it is expected that the child will first look for how many groups of 2 chocolate bars there are in 6 chocolate bars (6/2 = 3) and then multiply that number by 7. In other words, it is expected that children will first do the operation where an integer is obtained. If children use one method, regardless of the numbers used in the problem, they must be using one representation of the problem, which is determining the method to solve it.

The Experiment

Subjects

The study was carried out with 36 children between the ages of 9 and 14 years (9 years N = 2, 10 years N = 11, 11 years N = 11, 12 years N = 9, 13 years N = 1, 14 years N = 2). There were 12 children from 5th, 6th & 7th grades of a public school of Puerto Rico. Students who participated in the study were of average academic performance according to school grades.

Procedures and materials

The experimenter worked with one child at a time. The session with each child lasted around half hour and was audio-taped. The child first solved a set of word problems. The problems were presented one by one to the child. The child read the problem and could go back to it whenever he wanted. If the method by which the child had solved the problem was not clear to the experimenter, the child was asked to explain the solution.

Once each problem was solved the experimenter read the problem and asked the child to repeat it. Repetitions, if not done by rote, are a way to restate the problem as the child understands it. So, children's repetition are a way to indentify children's representations of the problems.

Once the problems were solved the child was asked to choose among a set of drawings the ones that represented the situation described in the problems. These drawings were presented to the children in the same order that they had been solved. Let us see in detail how these different tasks were carried out in the session that was held with each child.

Task I: The Problems

In the first task given in the session the child worked three two-step problems of the type described above and a one-step problem which also involved the notion of ratio such as the one contained in the following example:

"In the store they sell 12 candies in each bag.
María bought 4 bags of candies.
How many candies did she buy?"

In addition there were two fillers: a simple addition and a subtraction problem.

The fillers and the one-step problem were the same for all the children. They were the second (an addition problem), the fourth (a subtraction problem) and the sixth (a one-step problem) problem.

The two-step problems varied both in context and in the numbers used. There were three contexts: lollypops/chocolate bars, handkerchief/$ and children/teacher; and three sets of numbers: Only a/b was an integer, only c/b was an integer: a/b and c/b were both integers (a, b, c refer to the model problem in page 2). Each child solved one problem with each

of the contexts, using in each context different type of numbers. The numbers were varied among the contexts for different children. The two-step problems were the first, the third and the fifth problem, but the order of each problem varied among the children.

Task II: Model Recognition

The children were asked to choose among a set of drawings the ones that represented the situation described in the two-step and in the one-step problems which they had just solved. Some of the drawings correctly represented the problems while others did not (see figure 1 and 2). These drawings had been designed on the basis of an analysis of possible representations, correct or incorrect, that the child could make of the problem. For this analysis we used drawings made by a group of children in a pilot study in which they freely made representations of these same problems.

Insert figure 1 around here

Insert figure 2 around here

The experimenter displayed the drawings in front of the child together with the word problem that they represented and told the child:

> "Among these drawings there are some which correctly represent this problem and some which don't. Please pick the correct ones and tell me why are they correct."

It is expected that the drawings that children pick reflect the way they are representing the problem.

Response Categories

Repetitions

The repetition of a problem was classified as correct if it contained all the information needed to solve the problem. If during the repetition a number was forgotten or changed; for example, if repeating the problem "12 candies in each bag and 4 bags of candies" the child said "10 candies in each bag and 4 bags of candies", the repetition was still classified as correct.

The incorrect repetitions can be classified in two broad categories: omission errors, in which the child leaves out one or both premises of the problem; and distortion errors, where some part of the problem is changed. The omission errors can then be subdivided according to the premises that was left out and the distortion errors according to the element that was changed and how it was changed.

Model Recognition

When picking the drawings the children could choose one or several drawings. The selection was rated correct if it included only the correct representations (one or several). Some children, besides picking the correct representation(s), picked also an "incorrect" one. Yet, in their justification for picking the "incorrect" drawing they gave an interpretation of the drawing

that made it a correct representation of the problem. For example, a girl (11 years, 6th grade) picked two correct representations (Figures 1c and k) and two "incorrect" ones (Figures 1f and g). When she explained why she had picked each drawing her explanations were correct. For example in explaining why she picked Figure 1f (an incorrect representation) she explained:

> "Here there are 15 lollypops for 3 chocolate bars and he bought 12 chocolate bars".

The cases where the children besides the correct representation(s) picked incorrect representation(s) but gave correct explanations to justify their choices were rated as correct.

RESULTS

One-step problem (12 candies / bag x 4 bags of candies)

Twenty-nine out of 36 children (80.5%) worked the one-step problem correctly.

In both the repetitions and the model recognition tasks there was a close relationship between being able to solve the problem correctly and being able to repeat or pick the correct representation of the problem. Thirty-three of a total of 35 times[1/] (94.2%) the problem and the repetition were both correct or both incorrect. In the model recognition this occurred 83 percent of the time.

The performance of the children who solved the problem incorrectly on the repetitions and model recognition tasks showed that the major source of difficulty in solving the one-step problem was related to the

ratio concept. In the repetition four (4) out of the five (5) errors were distortion errors where a change from a ratio to an extensive quantity was made (e.g. from "12 candies per bag" to "12 candies" or "12 bags"). In the model recognition tasks the major type of error (57%) consisted in picking the cards where the relationship expressed in the ratio was not represented. These results confirm the results of Quintero (1980, 1981).

Two-step problems (e.g. 7 lollypops/2 chocolate bars, 6 chocolate bars)

Performance

The performance of the children on the two-step problems was significantly (Chi Square Test $p \leq .001$) lower than their performance on the one-step problem.

There were no significant differences on the performance of the children on the problems of different context or on the problems of different numbers (see table 1).

Insert Table 1 about here

Correct solution

The problems that were solved correctly were always solved by looking how many b were in c (this was sometimes done by direct division, other times by adding b until it gets to c) and then multiplying that quantity by a (or adding a that number of time). This led to the result that the problem where only a/b was an integer was sometimes solved by approximation. For example, there was a girl (11 years, 6th grade) who when working the problem:

> In a store they give 6 lollypops for every 2 chocolate bars.
> John bought 7 chocolate bars.
> How many lollypops were given to John?

said:

"2 chocolate bars----------6 lollypops
4 chocolate bars----------12 lollypops
6 chocolate bars----------18 lollypops

So it will be around 20 lollypops."

Half of the correct solutions of the problem where only a/b was an integer involved some type of approximation.

Incorrect solution

Children made different types of errors when working the problems incorrectly. Among them the more frequent ones were the following:

(1) Multiplying two of the quantities of the problem (41% of the errors)

Example: In the problem "6 lollypops for 2 chocolate bars, and bought 7 chocolate bars" they would give as answer 42 = 6 x 7. When children did this type of error they usually (58% of the time) multiplied a x c; (in the above example 6 x 7).

(2) Asserting that they didn't understand; (17% of the errors).

(3) Adding all three, or two of the quantities in the problem; (16% of the errors).

Repetition

Relation between repetition and performance

The relationship between being able to solve the problem correctly and being able to repeat the problem correctly is not

as strong in this case as in the one-step problem. Sixty-nine times out of 103 (66%) the problem and the repetition were both correct or both incorrect. A considerable number of time (32%), children worked the problem incorrectly but could repeat it correctly.

Type of error in the repetition

Distortion errors were more common than omission error (see Table 2). More important, out of the distortion errors, 23 (74%) consisted in a change in the ratio. There were two ways in which the ratio was changed. In one case it was changed from a ratio to an extensive quantity; for example, from "5 lollypops for every 4 chocolate bars" to "5 lollypops". In the other case, the ratio was changed to a per unit ratio; for example, from "5 lollypops for every 4 chocolate bars" to "5 lollypops for each chocolate bar".

Insert Table 2 about here.

Model Recognition

Relation between model recognition and performance

Eighty-seven percent of the times, solving the problem and picking drawings representing the problem were both correct or both incorrect. In some cases (8.3% of the times) children who had solved the problem incorrectly picked the correct representation of the problem; in other cases (4.6% of the times) children who had solved the problem correctly picked an incorrect representation.

Type of errors

The errors that children made while picking the drawings can be divided into several categories:

(1) Choosing cards where one or both of the elements mentioned

in the problem were present but not the relationships (39% of the time) (see Figures 1 f, g, h, i).

Some children in these groups picked just one card; others, picked all the cards in which there were a total of elements mentioned in the problem; for example, in the problem "15 lollypops for 3 chocolate bars and bought 12 chocolate bars" they picked all cards with a total of 15 lollypops.

(2) Changing the ratio to a per unit ratio (10% of the times) (see Figures 1 d, e).

(3) Picking all or some of the cards in which the ratio given in the problem was correct, but not the new situation that emerged given the new quantity (31% of the times) (see Figures 1 a, b).

Among this group 32 percent of the times the children picked only the card that showed the ratio alone (see Figure 1 a); 64 percent of the times they picked all the cards which showed the correct ratio, among them the correct card; and 4 percent of the times they picked just the card showing the correct ratio but not the new situation (see Figure 1b).

(4) Picking one card representing the first premise and another one representing the second one (12.5% of the times).

(5) Picking several cards and giving different reasons for each one. Among this group there may have been children who used the criteria of category (4) but didn't express it clearly (10% of the times).

Discussion

The great majority of the children had difficulty with the two-step problems. The difficulties, however, did not stem from the same source.

There was a group of children who had difficulty in understanding the concepts and relationships involved in the problem. The nature of this misunderstanding was not the same for all the children.

A group of children had difficulty in understanding the ratio given in the problem. In the repetitions 74 percent of the distortion errors consisted of repeating the ratio incorrectly. In the model recognition task 49 percent of the errors were in representing the ratio. Children varied in the way they represented the ratio. Some children in both the repetition and drawings changed the ratio to an extensive quantity. This group of children made the same type of error in solving the two-step problems; they added all three or two of the quantities. This group also coincided with the children who worked the one-step problem incorrectly. Another group of the children changed the ratio of the problem to a per unit ratio. The major type of error that these children made when solving the two-step problems was to multiply two of the quantities. The fact that most of the time they multiplied a x c adds evidence to the hypothesis that they are representing the ratio as a per unit ratio. In other words, given the problem; "6 lollypops for 2 chocolate bars and bought 7 chocolate bars" they interpreted it as:

"6 lollypops for each chocolate bars and bought 7 chocolate bars".

All the children in this group, except one, had worked the one-step problem correctly.

A group of children repeated correctly the ratio involved in the problem and picked the cards where this ratio was represented correctly, but they had difficulty representing the situation that emerged in the problem in both the repetitions and the drawings. When repeating the problem the main type of error that this group of children did was to change the extensive quantity in the problem from the element corresponding to the denominator of the ratio, to the element corresponding to the numerator of the ratio; for example, they repeated the problem "6 lollypops for every 2 chocolate bars, and bought 7 chocolate bars" to "6 lollypops for every 2 chocolate bars and bought 7 lollypops".

In the model recognition task they picked the cards in which the ratio given in the problem was represented but not the new situation that emerged given the new quantity.

This group of children seems to understand the relationship expressed in the ratio, but they have difficulties finding two new quantities in the same proportion.

There seems to be, then, various stages in the understanding of the concepts and relationships involved in the two-step problems:

(1) First, children understand per unit ratio. In this stage children can solve the one-step problem but not the two-step problems.

(2) Children understand ratios with "denominators" different from one, but given a ratio they have difficulty finding an equivalent ratio.

(3) Children can find equivalent ratios.

Once children get to the third stage in the above description they usually (61% of the times) solved the two-step problems correctly. Yet, there was a group of children who seem to be able to find equivalent ratio since they choose the correct drawing but solved the problem incorrectly (39% of the times).

This latter group of children worked the two-step problem using a single arithmetic operation. In all but one case the operation was multiplication. They are probably having difficulty with the method of solving two-step problems. Their difficulty may be due to various factors. Children could be having difficulty in organizing the information and/or planning a solution; or given their experience solving word problems in school, where usually they are one-step problems, children could interpret that problem-solving simply involves choosing the appropiate arithmetic operation. In fact a child when solving a two-step problem said "I don't know if I have to add, multiply, or divide".

General conclusions

This study shows that a major source of difficulty in solving two-step problems of a given structure was understanding the concepts and relationships involved in the problem. There is also evidence that there are levels of understanding of these concepts and relationships. There is need, however, to study further the different levels

of understanding of the ratio concept and how they affect the way the children solve this type of problems.

The fact that the concept of ratio, which is not common to all types of word problems, was the main source of difficulty in solving the problems shows the importance of having precise categories to characterize the word problems that children are asked to solve. If this work had studied children's difficulties solving word problems without looking at the different situations which they model it may have overlooked children's difficulty understanding ratios. The difficulty children will have with other type of problems, such as: "Jim has 3 pants, 5 shirts and 4 hats. How many outfits can he make?"; is of another nature.

The study points to the possibility that the method of solving two-step problems is also a source of difficulty. Yet, it is not clear from this study the specific methodological difficulties that children are having. It is, then important to explore the nature of the difficulties introduced by two step problems. For such a study problems should be designed using very simple concepts. This way the conceptual difficulties are minimized and more attention can be given to the difficulties caused by being two-step problems.

Studying these issues, the role of being a two-step problem and the levels of understanding of a ratio, not only will permit us to understand better children's difficulty with word problems but will allow us to design strategies for helping children surmount these difficulties.

Note

1/ A child is not indcluded because it was not clear if the repetition was correct or incorrect.

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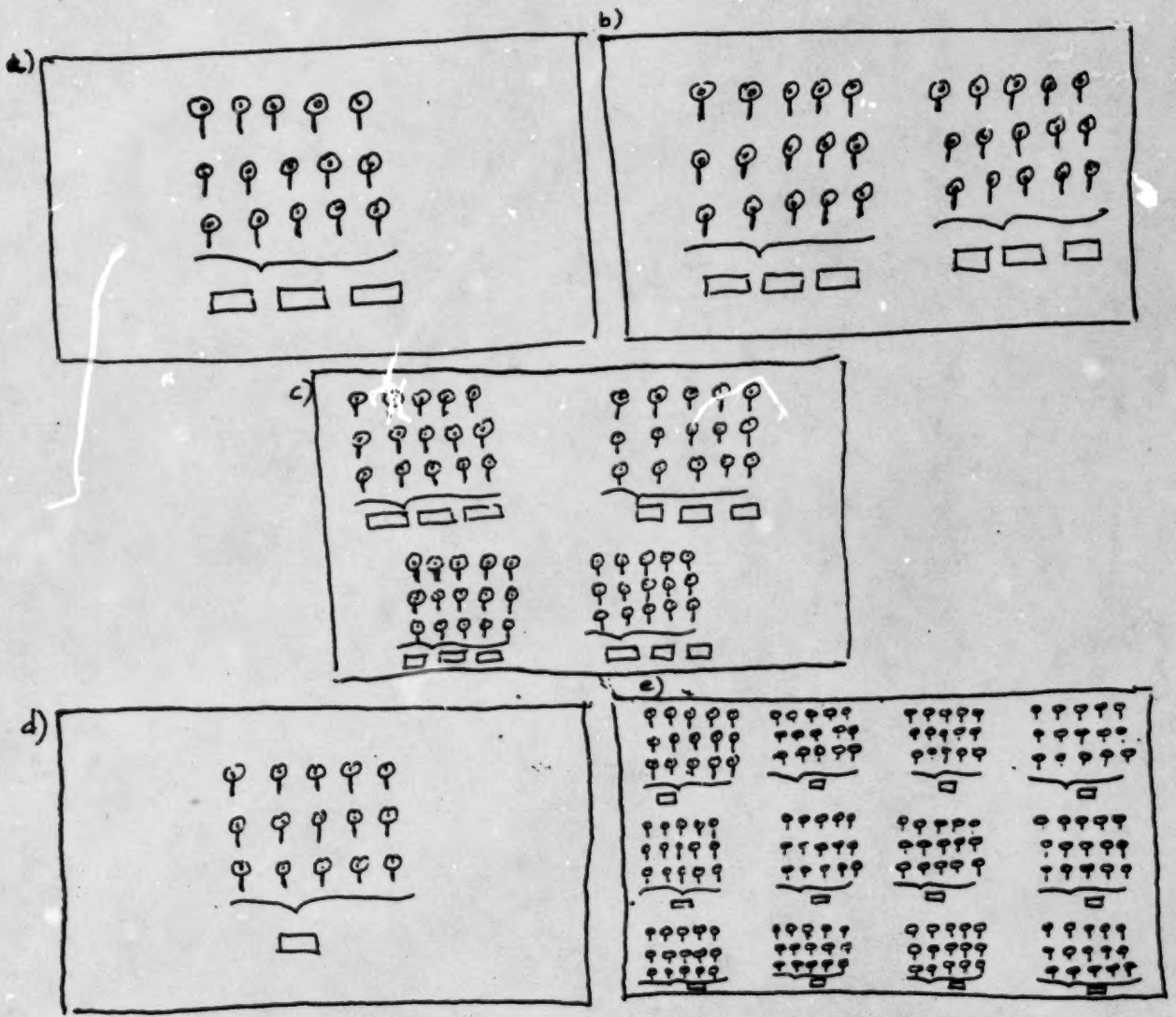
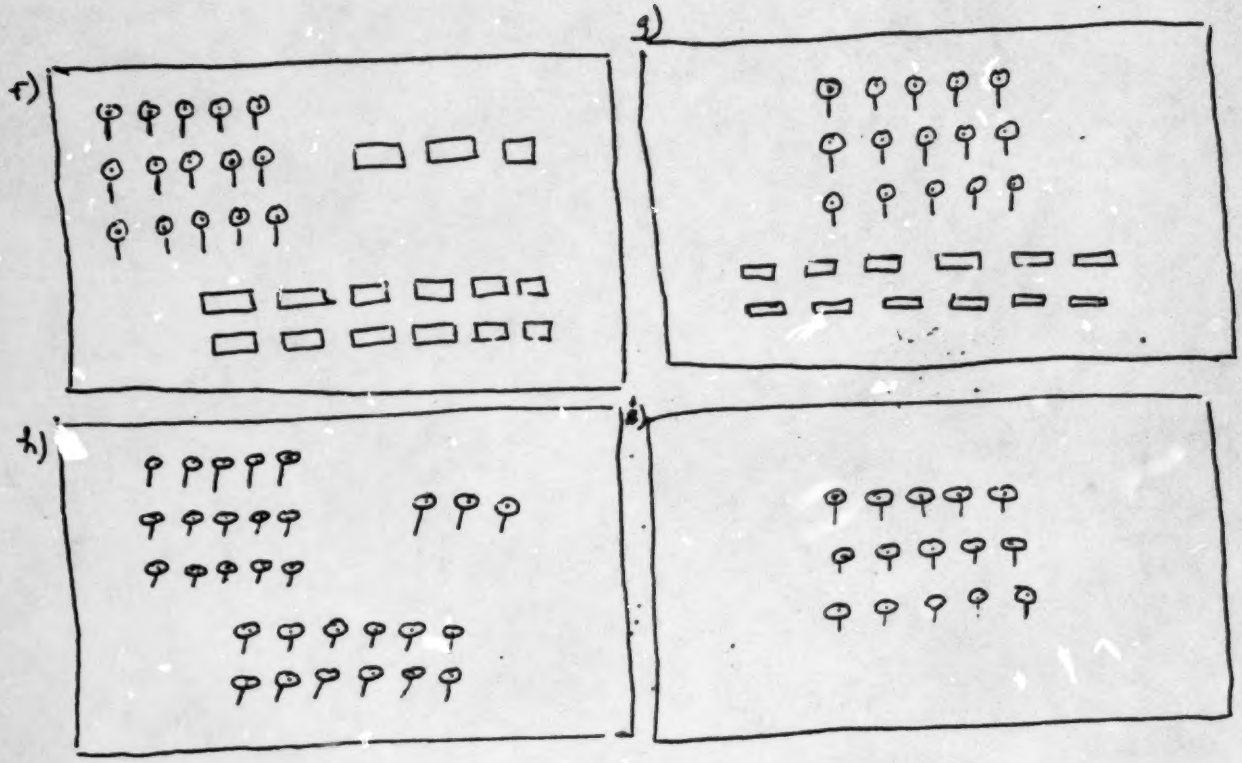
Figure 1: Drawing for the Model Recognition Task Two-Step Problem

Problem : In a store they gave 15 lollypops for every 3 chocolate bars that one buys.
John bought 12 chocolate bars
How many lollypops were given to John?




f)
g)
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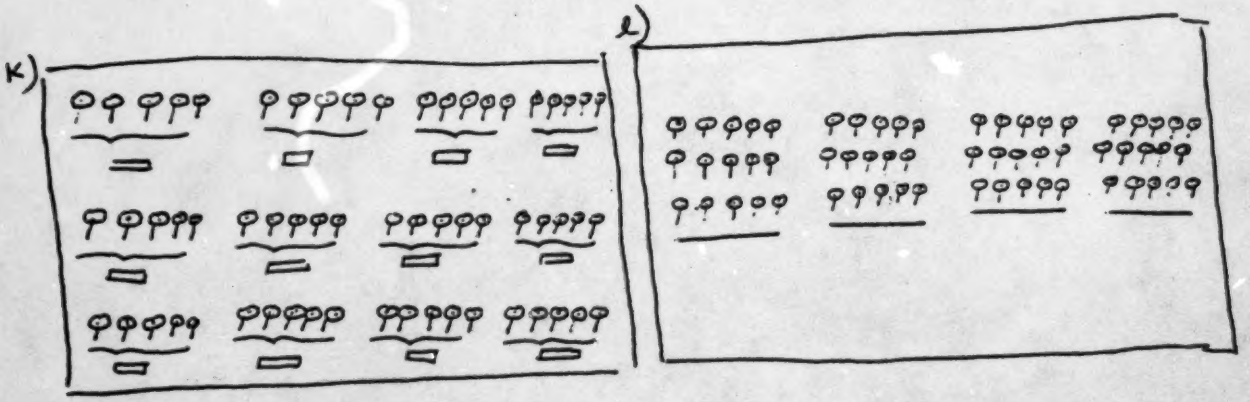
k)
l)

Figure 2: Drawings for the Model Recognition Task - One-Step Problem

Problem : In a store they sell 12 candies in each box.
María bought 4 boxes of candies
How many candies will María have?

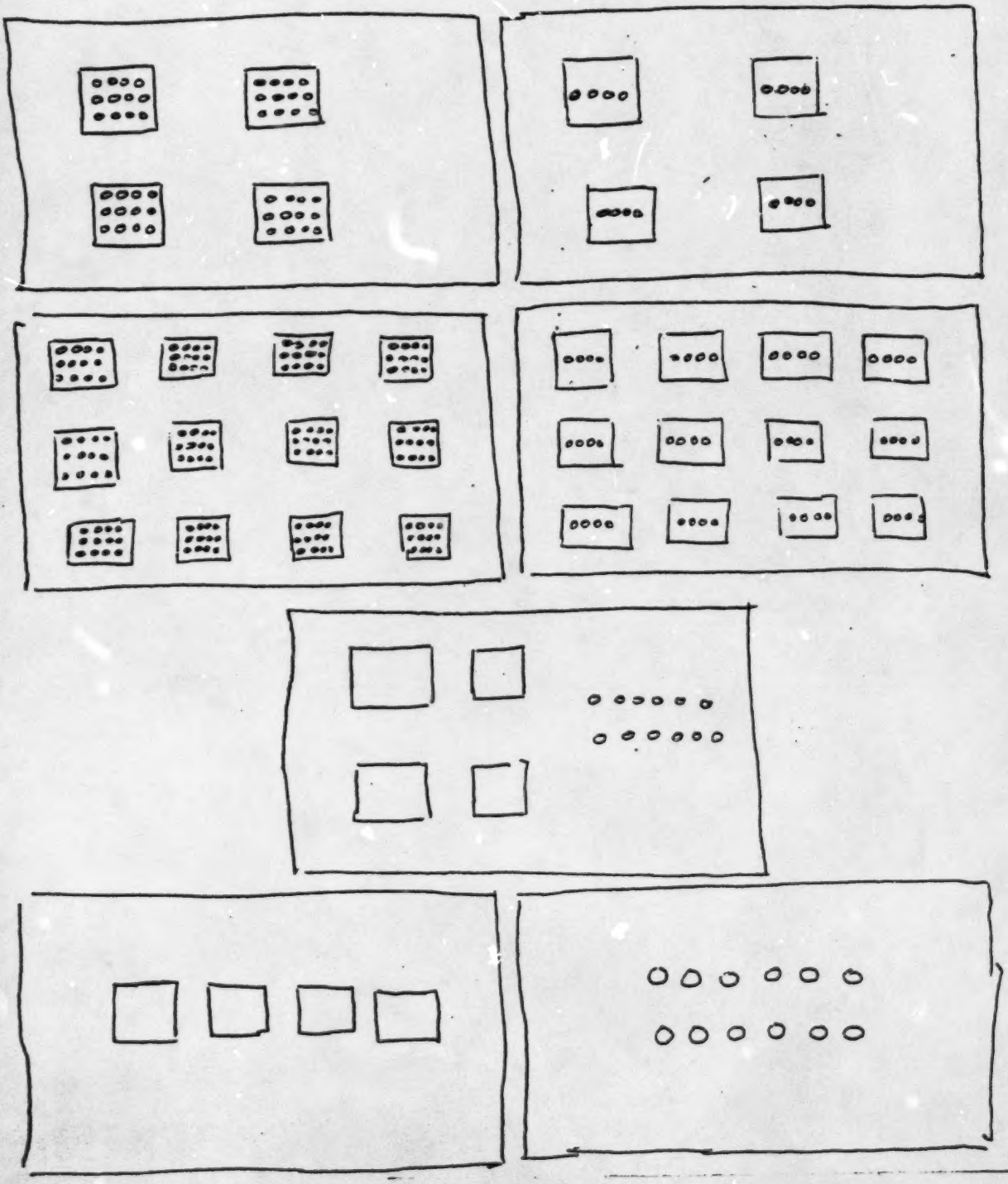

Table 1

Performance two-step problems

Problems by context			Problems by numbers		
	Correct	Incorrect		Correct	Incorrect
lollypops/choc. bars	5	31	a/b an integer	6	30
handkerchief/$	8	28	b/c an integer	6	30
children/teacher	8	28	Both integer	9	27

Table 2

Type of error when repeating the two-step problems

	Type of error	Number of times
Omission errors	Leave out premise involving the ratio	3
	Leave out premise involving the extensive quantity	5
	Leave out both premises	6
	Total	14
Distortion errors	Changes Ratio to an Extensive Quantity	8
	Changes x/y to x/1	15
	Ratio correct, changes extensive quantity	8
	Total	31